Can Quantum Gravitational Effects Manifest themselves at Large Distances?

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Abstract

Consider a proposed model of the universe with $\hbar$ much greater than its well-known value of $10^{-34} Js$. In this model universe, very large objects can show quantum behaviors. In a scenario with large extra dimensions, $\hbar$ can attains very large values depending on the dimensionality of spacetime. In this letter, we show that although conventional thinking indicates that quantum gravitational effects should manifest themselves only at very small scales, in actuality quantum gravitational effects can manifest themselves at large scales too. We use the generalized uncertainty principle with a non-zero minimal uncertainty in momentum as our primary input to construct a mathematical framework for our proposal.

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1 Motivation

The realm of quantum theory is very small scale, which usually consists atomic and subatomic levels. Gravity is a long range interaction and acts on ordinary scales as well as very large scales such as stellar and galactic ones. Usually we expect that quantum effects manifest themselves at very small scales. It is hard to accept that gravity as an interaction governing the large scale structures, can show quantum behavior at large distances. Here we are going to show that actually, gravity can show quantum behavior in large distances.

This idea is in contradiction with classical belief which accepts that quantum effects manifest themselves at very small scales. Recently, *El Naschie* has proposed a framework for investigation of such a fundamental problem[1]. He has indicated that " There remains the major problem of relating a theory developed for cosmological scales to problems on the scale of high energy elementary particles with wave particle dualism, tunneling effects and quantum entanglement." He then provides an elegant discussion of the problem using T-duality in E-infinity space. In our opinion, the formalism presented here will provide a suitable framework to deal with such a fundamental problem. To formulate our idea, we use extra dimensional scenarios and also the generalized uncertainty principle which consists of an absolute non-zero minimum uncertainty in momentum. We provide some clarifying examples to show in what extent the objects can manifest quantum effects under the given conditions.

2 An Effective $\hbar$

We begin our argument with the following question: can $\hbar$ be varying? To answer this question we consider the following generalized uncertainty relation which has been motivated from string theory and other approaches to quantum gravity[2-4]

$$\Delta x \Delta p \geq \hbar \Big(1 + \alpha^2 l_p^2 \frac{(\Delta p)^2}{\hbar^2}\Big), \tag{1}$$

where we have considered the absolute minimum of uncertainties to avoid the appearance of expectation values in the right hand side. The second term on the right hand side has its origin on the quantum effects of gravitation. If one tries to set a high precision position measurement of an electron, he should use very energetic photons. But in this situation one has to consider the gravitational interaction of photon and electron. In fact, a high energy photon causes spacetime fluctuation. Taking this point into account, one finds the second term of the right hand side as an extra uncertainty due to gravitational interaction of photon and electron. As a result, this generalized uncertainty relation consists of a natural cut off on the order of Planck length. Note that α is a string theory parameter of the order unity. Comparison with standard uncertainty relation $\Delta x \Delta p \geq \hbar$ shows that we can define

$$\hbar_{eff} = \hbar \Big(1 + \alpha^2 l_p^2 \frac{(\Delta p)^2}{\hbar^2}\Big). \tag{2}$$

The extra term on the right hand side of (1) is important at high momentum regime, where one can write approximately $\Delta p \sim p$. Therefore, one can obtain the following generalization

$$\hbar_{eff} = \hbar\left(1 + \alpha^2 l_p^2 \frac{p^2}{\hbar^2}\right). \tag{3}$$

In ordinary quantum mechanics, de Broglie principle is given by $\bar{\lambda} = \frac{\hbar}{p}$. Now equation (3) can lead to the following generalization of de Broglie principle

$$\bar{\lambda}_{eff} = \frac{\hbar_{eff}}{p} = \frac{\hbar}{p}\left(1 + \alpha^2 l_p^2 \frac{p^2}{\hbar^2}\right). \tag{4}$$

This generalization will affect fundamental arguments of quantum theory. For example, the entire argument of wave mechanics such as wave propagation, wave broadening and other domains such as coherent states of quantum mechanical systems should be re-examined within this framework[5-7]. Note that equation (4) can be interpreted in another fashion. We can write it in the following form

$$\bar{\lambda}_{eff} = \frac{\hbar}{p_{eff}} = \frac{\hbar}{p}(1 + \alpha^2 l_p^2 \frac{p^2}{\hbar^2}\Big). \tag{5}$$

This point of view leads to "modified dispersion relation" which can be written as follows

$$\frac{1}{p_{eff}} = \frac{1}{p}\left(1 + \alpha^2 l_p^2 \frac{p^2}{\hbar^2}\right). \tag{6}$$

Modified dispersion relations have some signature in ultra-high cosmic ray showers which are under serious investigation[8].

Up to this point, we have been satisfied that $\hbar$ can be varying with momentum. This idea has its very basic notion in the string theoretical considerations.

3 $\hbar$ in Models with Extra Dimensions

In this section, we argue that in scenarios with extra dimensions[9,10], $\hbar$ can attains very large values relative to its 4-dimensional counterpart. In 4-dimensions, one can write

$$m_p l_p = \hbar,$$

while in $4 + d$ dimensions(we use Arkani-Hamed, Dimopoulos and Dvali model of extra dimensions[9]) this statement generalizes to

$$M_f L_f = \hbar,$$

where M_f and L_f are Planck mass and Planck length in model universe with extra dimensions respectively[11]. The relation between 4-dimensional Planck length and the extension of extra dimensions R(we suppose all extra dimensions have the same extension) is given by

$$m_p^2 = M_f^{2+d} R^d. \tag{7}$$

Therefore, we can write

$$M_f = \frac{\hbar}{L_f} = \frac{\hbar_{eff}}{l_p} = \frac{\hbar_{eff}}{\hbar} m_p \quad \Longrightarrow \quad M_f^2 = (\frac{\hbar_{eff}}{\hbar})^2 M_f^{2+d} R^d. \tag{8}$$

This relation can led us to the following result

$$\hbar_{eff} = \frac{\hbar}{(M_f R)^{\frac{d}{2}}}. \tag{9}$$

The extension(radius) R of these extra dimensions, for $M_f \sim TeV$, typically lies in the range from $1mm$ to $10^3 fm$ for d from 2 to 7. Suppose the case with $d = 3$, that is, with only three extra dimensions. In this case $R = 10nm = 10^{-8}m$. With $M_f = 1TeV = 1.6 \times 10^{-7}J$, we find

$$\hbar_{eff} \sim 10^{-11} Js.$$

In a model universe with this value of Planck constant, de Broglie relation $\lambda_{eff} = \frac{\hbar_{eff}}{mv}$ shows that a particle with detectable wave properties and with usual speed can have very large mass. For Example, suppose that $\lambda = 10^{-14}m$ which is in the range of detectors resolution and $v = 100m/s$. We find $m = 10kg$. Such a large mass has quantum effects in a universe with $\hbar_{eff} \sim 10^{-11} Js$.

Thus far, we have been satisfied that quantum effects are not restricted to very small masses depending on the value of $\hbar$. In the next section we show that gravity can manifest quantum behavior in large distances.

4 Quantum Gravity Effects in Large Distances

Spacetime at short distances has a noncommutative structure. This noncommutativity can be addressed in the generalized uncertainty principle. Recently we have shown that spacetime noncommutativity and the generalized uncertainty principle when are applied to the issue of black hole thermodynamics, give the same results[12]. Since uncertainty principle is the foundation of quantum theory and gravity induces uncertainty, one has to incorporate gravitational uncertainty from the very beginning of the quantum theory formulation. There are several possibilities to write these generalized uncertainty principles. The most general form of generalized uncertainty principle can be expressed as[13]

$$\Delta x \Delta p \geq \hbar\Big(1+\beta^2\frac{(\Delta x)^2}{l_p^2}+\alpha^2 l_p^2\frac{(\Delta p)^2}{\hbar^2}+\gamma\Big). \tag{10}$$

Since we are dealing with absolute minimum of uncertainties, we suppose that $\gamma = 0$. The resulting generalized uncertainty principle has minimal non-zero uncertainty both in position and momentum. One possible choice is the case where $\alpha = 0$. Then we find

$$\Delta x \Delta p \geq \hbar\Big(1+\beta^2\frac{(\Delta x)^2}{l_p^2}\Big). \tag{11}$$

In this case we have non-zero minimal uncertainty only in momentum. Quantum gravitational effects are important only in the limit where the second term in the right hand side of (11) becomes comparable with the first term, that is where

$$\beta\frac{\Delta x}{l_p} \approx 1 \Longrightarrow \Delta x \approx \frac{l_p}{\beta}. \tag{12}$$

Since usually $\beta \ll 1$, this statement shows that quantum gravitational effects can manifest themselves in very large distances. Physically this is the case since generalized uncertainty principle in the form of (11) contains quantum gravitational effects in the limit of high Δx. Therefore, we conclude that quantum effects of gravitation can be revealed in large distances as well as very small distances. Note that in E-infinity theory of *El Naschie* such a result can be obtain by admitting and tuning of dimensions in a concrete way. A similar argumentation but without relating its to extra dimensional scenarios has been pointed in [14]. It seems that there is a correspondence between large/small distance behavior of quantum gravitational effects. This idea can be examined in the next generation of accelerators, specially the Large Hadronic Colider(LHC)[15].

5 Summary

In this letter we have argued that quantum gravitational effects can manifest themselves at large distances as well as small distances. We have found a generalization of $\hbar$ and also de Broglie principle to show the possibility of having quantum effects for large masses. We have used string theory generalized uncertainty principle and ADD scenario of extra dimensions for formulation of our idea. Our proposal has the ability to be examined in the next generation of Hadronic Coliders at CERN. Actually, possible verification of extra dimensions proposed by ADD model and also possible detection of TeV black hole remnants in LHC and ILC will provide indirect experimental test of our conjecture[16,17].

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